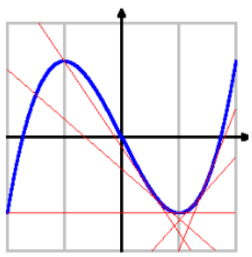
3.4 Concavity and the Second Derivative Test

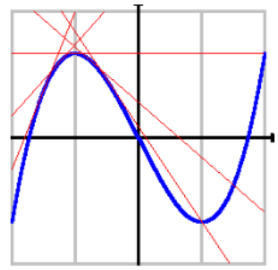
Concave up like a cup…concave down like a frown!

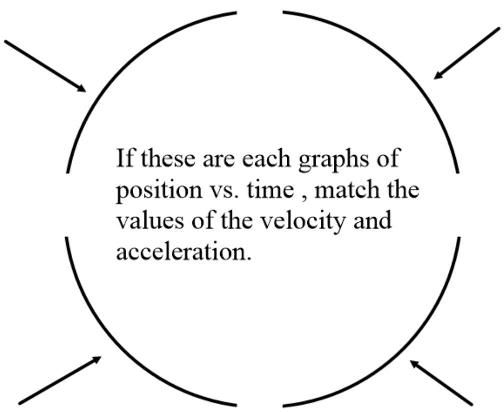
Definitions:

Concave up:



Concave down:





Concavity test

A function is concave up where:

A function is concave down where:

Point of Inflection(POI) is:

Ex 1. Determine the concavity and identify any points of inflection of $f(x)=\frac{2}{x}+\sqrt{x}$

The second derivative test:

1.

2.

Ex 2. Find the relative extrema and inflection points, intervals of concavity and increasing and decreasing. $f(x) = 5 + 3x^2 - x^3$

EX 3. A function f is cont's on [-3,3] and its first and second derivatives are as follows:

x	(-3,-1)	-1	(-1,0)	0	(0,1)	1	(1,3)
$f'(x)$	Positive	0	Negative	Negative	Negative	0	Negative
$f''(x)$	Negative	Negative	Negative	0	Positive	0	Negative

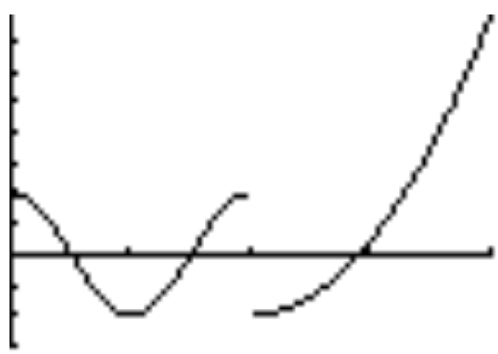
At what x values does f have...

a. relative minima? Justify.

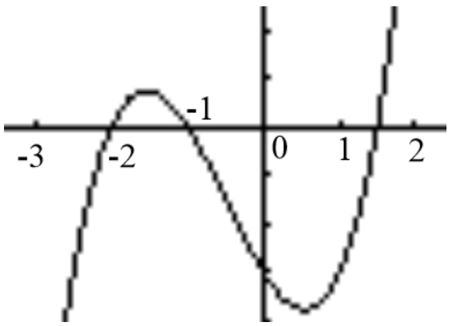
b. relative maxima? Justify.

c. points of inflection? Justify.

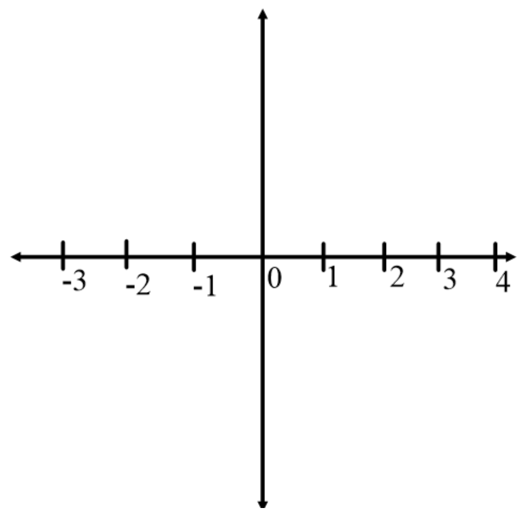
Ex 4. Function f(x) is graphed below and is defined on [0,4]. Estimate the intervals on which f '(x) is positive or negative and on which intervals f "(x) is positive or negative.



ex 5. For the graph shown, at which integer value of x is it true that both f '(x)>0 and f "(x)>0?



ex 6. Sketch the graph of a continuous function from [-3,4] which satisfies all the following conditions:
f '(x)<0 for all real numbers x≠1;
f '(1) does not exist;
f "(x)<0 for all x<1; and
f "(x)>0 for all x>1



Ex 7. Use f' and f'' to graph a possible f(x).

$f'(x) = 4x^3 - 12x^2$